A SOLUTION OF THE GENERATION PUZZLE FROM YANG-MILLS DUALITY

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A solution to the generation puzzle based on a nonabelian generalization of electric-magnetic duality is briefly reviewed. It predicts 3 and only 3 generations of fermions and explains the hierarchical mass spectrum as well as the main features in both the quark and lepton mixing matrices. A calculation to leading perturbative order already gives reasonable values to about half of the Standard Model parameters.

The complex of questions known loosely as the "generation puzzle" has puzzled physcists for well over a generation. In practical terms, it accounts for more than two-thirds of the twenty-odd empirical parameters needed to specify the Standard Model, and is thus rightly regarded by many as one of the most important and urgent problems facing particle physics today. In this talk, we review briefly a possible solution to the puzzle based on a nonabelian generalization to electric-magnetic duality known in the literature as the DSM scheme. For references see the list in [I], hep-th/0007016, to which the citation numbers in this paper refer.

Let us first recall the main facts involved.

First, the matter in our universe is made from fundamental fermions of 4 species differing in colour or weak isospin, namely U-type quarks, D-type quarks, charged leptons, and neutrinos. However, for no known theoretical reason, there are 3 generations to each species having the same quantum numbers except for their masses.

Secondly, the masses of the 3 generations follow a markedly "hierarchical" pattern. For the three charged species, the masses in MeV are roughly[1]:

$$\begin{pmatrix} m_t \\ m_c \\ m_u \end{pmatrix} = \begin{pmatrix} 180000 \\ 1200 \\ 4 \end{pmatrix}; \begin{pmatrix} m_b \\ m_s \\ m_d \end{pmatrix} = \begin{pmatrix} 4200 \\ 120 \\ 7 \end{pmatrix}; \begin{pmatrix} m_\tau \\ m_\mu \\ m_e \end{pmatrix} = \begin{pmatrix} 1777 \\ 105 \\ .5 \end{pmatrix}, \qquad (1)$$

where it is seen that the mass drops by one to two orders of magnitude from generation to generation.

Thirdly, the state vectors of the 3 generations are nearly but not exactly aligned between the up- and down-states. The unitary matrix giving the relative orientation of the down-triad to the up-triad is known as the CKM matrix for quarks and the MNS matrix for leptons, and present experiments give for the absolute values of the matrix elements[1]:

$$|V_{CKM}| = \begin{pmatrix} 0.975 & 0.220 & 0.003 \\ 0.220 & 0.974 & 0.04 \\ 0.008 & 0.04 & 0.999 \end{pmatrix}, \tag{2}$$

$$|U_{MNS}| = \begin{pmatrix} ? & 0.4-0.7 & 0.0-0.15 \\ ? & ? & 0.45-0.85 \\ ? & ? & ? \end{pmatrix}. \tag{3}$$

The matrix for quarks is close to but definitely not the identity, with the nonzero off-diagonal elements representing the rates of some very well measured hadronic proscesses. Whereas for the leptons, the matrix is far from diagonal with the large off-diagonal elements representing the results from some recent experiments on neutrino oscillations[2,3].

By the generation puzzle, one means then not only the mystery why there should be 3 and apparently only 3 generations of fundamental fermions, but also why their masses and mixings should fall into such peculiar patterns. In current formulations of the Standard Model, these features are taken for granted while the masses in (1) as well as the entries in (2) and (3) have all to be supplied from experiment.

Let us see now how duality helps to answer these questions.

First, the fact that there are 3 generations of fermions with very similar properties suggests the existence of an underlying 3-fold symmetry. This new symmetry has to be broken since the 3 generations have different masses. The beauty of the DSM scheme is in suggesting a natural candidate for this generation symmetry, as follows. Electromagnetism is symmetric under the Hodge star operation: ${}^*F_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, which interchanges electricity and magnetism. This implies in particular that the theory is invariant under a doubled gauge symmetry $U(1) \times \tilde{U}(1)$, where the first $U(1)$ represents the original (electric) gauge group while the second $\tilde{U}(1)$ represents its dual (magnetic) counterpart. In terms of $U(1)$, electric charges are sources and magnetic charges are monopoles, but in terms of $\tilde{U}(1)$ these charges appear instead as monopoles and sources respectively. What happens in nonabelian Yang-Mills fields? It was known that under the Hodge star operation, Yang-Mills theory is not symmetric[4]. However, it was shown that if the Hodge star is replaced by a certain generalized dual transform[5], then duality is recovered[a]. This means that the theory is again invariant under a doubled gauge symmetry $SU(N) \times \widetilde{SU}(N)$. In particular, for colour, one has $SU(3) \times \widetilde{SU}(3)$. Furthermore, it was shown[15] using a result of 't Hooft[14] that given the confinement of colour $SU(3)$, the dual symmetry $\widetilde{SU}(3)$ is broken, so that within the colour theory itself, there is already a broken 3-fold symmetry ready to play the role of the generation symmetry. Since this symmetry already exists and needs to be physically accounted for in any case, it seems natural to identify it with the generation symmetry[16]. As a result, one concludes that there are 3 and only 3 generations of fermions labelled by the 3 different colour magnetic charges, offering thus an answer to the leading question of the generation puzzle.

But why should the mass spectrum of fermions be hierarchical? To answer this, one needs to know how the $\widetilde{SU}(3)$ dual colour symmetry is broken which information is not given by[14]. Fortunately, the theoretical framework developed for Yang-Mills duality[5] already offers candidates for the Higgs fields required in the form of the frame vectors (complex dreibeins) $\phi_a^{(a)}; a, (a) = 1, 2, 3$ in dual colour space, which seem to play dynamical roles[13] as physical fields, and are $\widetilde{SU}(3)$ triplets, space-time scalars with finite classical lengths. With these as Higgs fields, the following Yukawa coupling is suggested:

$$\sum_{(a)[b]} Y_{[b]} \bar{\psi}_L^a \phi_a^{(a)} \psi_R^{[b]}, \tag{4}$$

where, as in electroweak theory, we have taken left-handed fermions in the fundamental representation, i.e. triplets, and right-handed fermions as singlets. In turn, the above Yukawa coupling implies a tree-level mass matrix of the following factorized form:

$$m = m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x, y, z), \tag{5}$$

where (x, y, z) is a normalized vector with its components given by the vacuum expectation values of the lengths of $\phi^{(a)}, (a) = 1, 2, 3$. It follows therefore that at tree-level, m has only one non-zero eigenvalue, offering thus the beginning of an anwer to the fermion mass hierarchy.

[a] The dual properties of Yang-Mills fields are of course a highly nontrivial theoretical subject. It has taken literally years to derive the cited result which involved a development with Polyakov's loop space techniques and other things too long to be reported here. For a brief outline of this work, the reader is referred to [I], and for more details to hep-th/9904102 and hep-th/0006178, and original references therein.

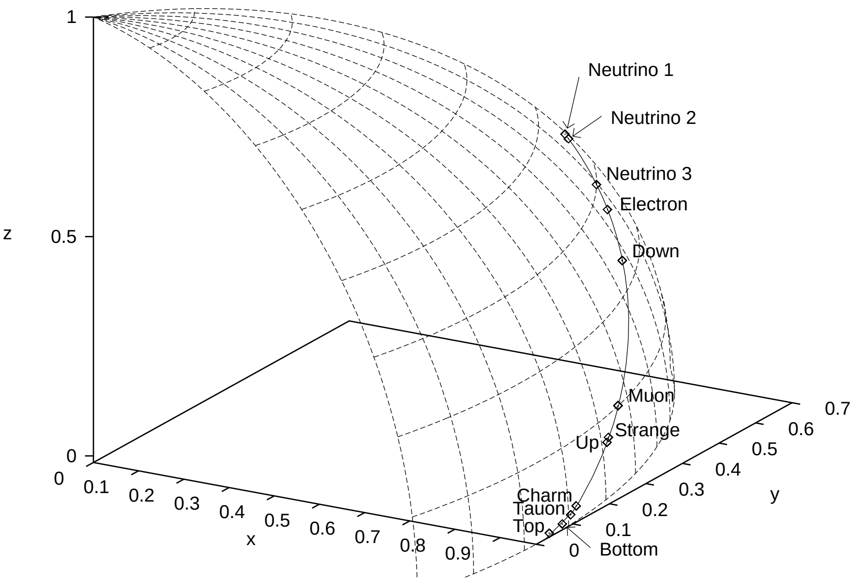

Figure 1. Trajectory traced out by the vector (x', y', z') in generation space

What happens to the mass matrix under radiative corrections? Like many other quantities in quantum field theory, the fermion mass matrix here runs with changes in scale, but because of the particular way (4) in which it is coupled, it remains factorized in form as in (5) only with the vector (x, y, z) now replaced by one, say (x', y', z'), which varies with the scale, tracing out with changing scales a trajectory on the unit sphere. The trajectory has a high energy fixed point at $(1, 0, 0)$ and a low energy fixed point at $\frac{1}{\sqrt{3}}(1, 1, 1)$, but its actual shape depends on the Higgs vev's (x, y, z), and the Yukawa couplings, which parameters have to be fitted to data. The result of our fit is shown in Figure 1 which coincides for all fermion species to a high accuracy[18,19]. It is remarkable that nearly all the previously noted features of the empirical mass spectrum and mixing matrices can now be read off from this figure.

To do so, one has first to clarify some points. The fact that the vector (x', y', z') varies in orientation (rotates) with changing scales means that state vector of physical fermion states each defined at its own mass scale, need no longer be eigenstates of the mass matrix at some other scales. Hence, despite the fact that the mass matrix remains factorizable at all scales, mass need no longer vanish for the two lower generations as at tree-level[16,20]. We say in this case that the mass "leaks" from the highest generation into the lower ones, giving them small but finite masses. Secondly, the state vectors for up- and down-states, being defined at different scales, need not now have the same orientation as they did at tree-level, resulting thus in nontrivial mixing matrices. Notice that both lower generation masses and mixing are here governed just by the speed at which the vector (x', y', z') rotates.

With these observations made, let us turn back to examine the mass spectrum in (1). One notices there not only that masses are dropping by large factors from generation to generation as expected from the leakage mechanism, but also that the drop factor is larger the heavier the mass, thus: $m_c/m_t < m_s/m_b < m_\mu/m_\tau$. This is easily understandable, for the heavier the mass, the nearer the state is to the high energy fixed point $(1, 0, 0)$ as seen in Figure 1, and hence the slower the rotation and the smaller the leakage.

Next, one sees that the fact mixing is generally smaller for quarks than for leptons in (2) and (3) is also easily understood. The separation between t and b on the trajectory in Figure 1 being so much smaller than that between τ and ν_3, there will be less rotation or disorientation between the up- and down-states for quarks than for leptons and hence smaller mixing also.

Finally, one sees that even the relative sizes of elements within the same mixing matrix, in particular, that the corner elements are exceptionally small in both (2) and (3), are readily understood by means of some simple differential geometry[21]. To a good approximation, the state vectors of the three generations can be represented as an orthonormal (Darboux) triad at the location of the heaviest generation, with the heaviest generation state as the radial vector to the sphere, the second generation state as the tangent vector to the trajectory, and the lightest generation state as the vector orthogonal to both the above. The mixing matrix then appears just as the matrix representing the rotation undergone by this triad as it is transported along the trajectory from the location of the heaviest up-state to the heaviest down-state. To leading order in the distance transported, elementary differential geometry[22] gives this rotation matrix as:

$$V_{CKM} \sim \begin{pmatrix} 1 & -\kappa_g \Delta s & -\tau_g \Delta s \\ \kappa_g \Delta s & 1 & \kappa_n \Delta s \\ \tau_g \Delta s & -\kappa_n \Delta s & 1 \end{pmatrix}, \tag{6}$$

with κ_n being the normal curvature, κ_g the geodesic curvature, and τ_g the geodesic torsion of a curve on a surface. For the unit sphere, $\kappa_n = 1$ and $\tau_g = 0$. From this we deduce first that the corner elements (13 and 31) are of second order in Δs and therefore small compared with the others, and secondly, that the 23 and 32 elements are given approximately just by

Table 1. Predicted CKM matrix elements for both quarks and leptons

$Quantity$	$Experimental Range$	$Predicted$ $Central Value$	$Predicted Range$
$\lvert V_{ud}\rvert$	$0.9745 - 0.9760$	0.9753	$0.9745 - 0.9762$
$\lvert V_{us}\rvert$	$0.217 - 0.224$	(0.2207)	$input$
$\lvert V_{ub}\rvert$	$0.0018 - 0.0045$	0.0045	$0.0043 - 0.0046$
$\lvert V_{cd}\rvert$	$0.217 - 0.224$	(0.2204)	$input$
$\lvert V_{cs}\rvert$	$0.9737 - 0.9753$	0.9745	$0.9733 - 0.9756$
$\lvert V_{cb}\rvert$	$0.036 - 0.042$	0.0426	$0.0354 - 0.0508$
$\lvert V_{td}\rvert$	$0.004 - 0.013$	0.0138	$0.0120 - 0.0157$
$\lvert V_{ts}\rvert$	$0.035 - 0.042$	0.0406	$0.0336 - 0.0486$
$\lvert V_{tb}\rvert$	$0.9991 - 0.9994$	0.9991	$0.9988 - 0.9994$
$\lvert V_{ub}/V_{cb}\rvert$	0.08 ± 0.02	0.1049	$0.0859 - 0.1266$
$\lvert V_{td}/V_{ts}\rvert$	< 0.27	0.3391	$0.3149 - 0.3668$
$\lvert V_{tb}^* V_{td}\rvert$	0.0084 ± 0.0018	0.0138	$0.0120 - 0.0156$
$\lvert U_{\mu 3}\rvert$	$0.56 - 0.83$	0.6658	$0.6528 - 0.6770$
$\lvert U_{e3}\rvert$	$0.00 - 0.15$	0.0678	$0.0632 - 0.0730$
$\lvert U_{e2}\rvert$	$0.4 - 0.7$	0.2266	$0.2042 - 0.2531$

the transportation distance Δs, namely for the quark case by the distance between the top and bottom quarks along the trajectory, and for the lepton case by the distance between τ and ν_3, which statement is valid to very good approximation in (2) and (3) as can be easily verified in Figure 1 with a piece of string.

Of course, having constructed a detailed scheme, one can go far beyond the qualitative discussions of the preceding paragraphs. For example, from the one-loop calculation reported in[20], one obtains the numbers given in Table 1, where one sees that all entries more or less overlap with the present experimental limits, apart from one exception U_{e2} which is particularly difficult for our calculation to get correct. And all these numbers have been obtained by adjusting only one parameter to the Cabibbo angle $V_{us} \sim V_{cd}$, the other two parameters in the scheme having already been fitted to fermion masses.

One concludes thus that the DSM scheme does offer to-date a viable solution to the generation puzzle. Besides, with all parameters fixed, it is now highly predictive with ramifications ranging from rare hadron decays[23] and $\mu - e$ conversion[27], to air showers[24] and fermion transmutation[29,30], and so far in all these the DSM has survived existing experimental tests.